Structural Performance of Reciprocal Structures formed by using Islamic Geometrical Patterns

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Many Islamic geometric patterns consist of stripes which are recognizable in the two dimensional patterns. These stripes systematically pass over or under each other, thus they create a tessellation. This system has the same principle with reciprocal frame structures. Considering this situation, in this study, it is aimed to lift the two dimensional Islamic geometric patterns to the third dimension with the principle of reciprocal frame structures. A selected Islamic geometric pattern has been lifted to the third dimension in the reciprocal structure principle, and structural analyzes have been performed.

Keywords: *Reciprocal frame structures, Islamic geometric patterns, Structural analysis*

INTRODUCTION

Reciprocal frame structures consist of mutually supported rigid elements. A module in reciprocal frame structure is a short piece of the whole structure (Popovic Larsen, 2008; Anastas et. al. 2016). Reciprocal frame structures are also known as Nexorades (Baverel, 2000).

Although it is an interesting type of structure, it is not widely used in construction industry today. Anastas et. al. (2016) explain the reason for this as the difficulty in adapting to the holistic form of the structure as it goes from module to global scale.

On the other hand, reciprocal frame structures have many advantages. Large openings can be covered via bringing together a large number of short pieces. Therefore, reciprocal frame structures are also known as one of the structure types used to create large spans in buildings. In addition, as Di Carlo (2008) mentioned, it can be suitable for emergency situations, since it can be built quickly with local materials. In addition, Popovic Larsen (2014) points out that the benefit of the reciprocal frame structure is that in symmetrical configurations, all joints and all members are identical. Moreover, Popovic Larsen (2019) mentions that the joints are reversible and allow disassembly and reusability, and reciprocal frame structure is lightweight and easy to carry, and in gridshell-like reciprocal frame structures, the failure of a piece or connection does not destroy the whole system.

In this study, it has been determined that Islamic geometrical patterns with polygonal system and reciprocal frame structures are constructed with the same principle. In the both system, the pieces of the patterns systematically overlap each other.

In Islamic geometric patterns, which are carved on stone and wood, there is a semi-three dimensionality, which is formed by passing the strips over each other. This situation can be observed at the works of many researchers who had conducted studies on Islamic geometrical patterns (Bonner, 2017; Agirbas, 2020). However, to date, there is no study on the potential relationship between reciprocal frame structures and Islamic geometric patterns with polygonal system. Therefore, in this study, it is aimed to lift the two dimensional Islamic geometric patterns to the third dimension with the principle of reciprocal frame structures and to compare them as means of structural performance. Thus, based on the traditional elements, contemporary reciprocal structure models with high structural performance can be obtained in a variety of different forms.

BACKGROUND

Reciprocal frame structures

It is known that reciprocal frame structures were used in the twelfth century in Chinese and Japanese architecture (Di Carlo, 2008). These structures, built using timber as a material, were specifically built as roof supporting systems, and are also known as mandala roof (Kohlhammer and Kotnik, 2011).

The works of Villard de Honnecourt (around 1240), Sebastiano Serlio, Leonardo da Vinci and John Wallis are known as the early examples of the reciprocal frame structures (Houlsby, 2014). There are notes about the use of multiple beams in the Leonardo's Codex Atlanticus fol. 899v, which includes reciprocal frame structure drawings (Williams, 2008). It is possible to group these early examples according to their construction type. Parigi and Pugnale (2012) mentions that Da Vinci used un-notched elements, Wallis used notched elements, and Serlio used aligned axes for constructing reciprocal frame structures, and relates the reasons of different usage of elements to their proximity to the plane (Figure 1). For example, in the version with the usage of the un-notched elements, a dome-like situation will inevitably occur.

It is possible to give contemporary examples of various reciprocal frame structures that have been constructed. For example, Archaeological Shelter in at Bibracte, France, built with the principle of Reciprocal Frame structures in 2008, was designed by architect Paul Andreu and engineers Bernard Vaudeville and Simon Aubry (Gelez et. al 2011). Frans Masereel Centrum in Kasterlee, Belgium was designed by architects LIST, Hideyuki Nakayama Architects and engineers Bollinger + Grohmann and Bureau Bouwtechniek, using reciprocal frame structures on the roof (Bergis and De Rycke, 2017). Mount Rokko-Shidare Observatory (Japan, 2010) was designed with the reciprocal frame structure principle by architect Hiroshi Sambuichi and Ove Arup and Partners (Popovic Larsen, 2014). Pizzigoni (2009) proposed fibre-reinforced concrete based reciprocal frame structure for Italian World Expo Pavilion in Shanghai 2010. Kreod Pavilion by Pavilion Architecture and Rambøll London (London, UK, 2012), Seiwa Bunraku Puppet Theater by Kazuhiro Ishii (Japan,1990s), Forest Park Pavilion by Shigeru Ban, Cecil Balmond and ARUP (St Louis, Missouri, 2007), Coca Cola Beatbox Pavilion by Asif Khan and Pernilla Ohrstedt (London, UK, 2012) can be given as other examples of buildings with reciprocal frame structures.

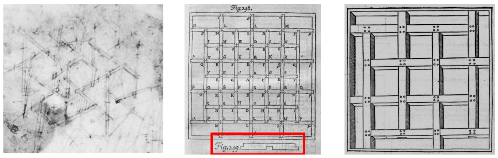


Figure 1
Un-notched elements by Da Vinci, notched elements by Wallis, aligned axis by Serlio (Parigi and Pugnale, 2012)

In addition, various researches are carried out on reciprocal frame structures. For example, Douthe and Baverel (2009) proposed a form-finding method for reciprocal frame structures by using dynamic relaxation algorithm. Parigi and Kirkegaard (2014) developed the Grasshopper plug-in, named Reciprocalizer, which can create various types of reciprocal frame structures. Thonnissen (2014) described a form-finding tool for reciprocal structures in his paper. Song et. al. (2014) proposed an interactive tool for designing reciprocal frame structures. Anastas et.

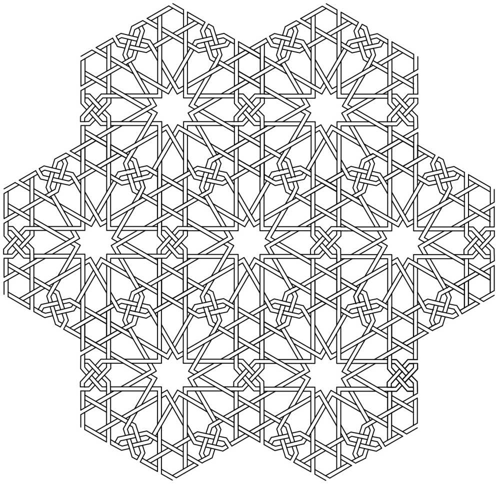


Figure 2
A tessellation in Ahlat cemetery (Agirbas, 2020)

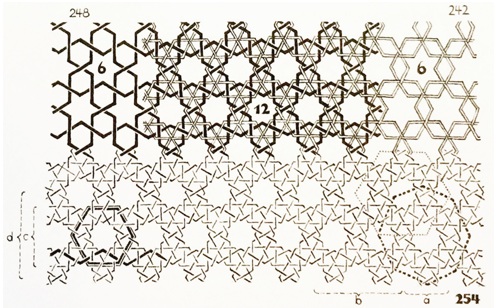


Figure 3
Example of Islamic geometric pattern (Schneider, 1980)

al (2016) developed a parametric model that allows the reciprocal frame pattern to be adapted to various surface forms. Gerber and Pantazis (2016) focussed on form-finding process of reciprocal frames which are informed by structural and environmental analyses. Moreover, Xian et. al. (2020) conducted a study on robotic fabrication of a double-curved reciprocal frame wall.

Potential of Islamic geometric patterns

Two-dimensional Islamic patterns are mathematically defined patterns. It can be easily seen that some of these two-dimensional patterns have overlaps that give a sense of third dimensionality. For example, as Agirbas (2020) mentioned, three-dimensionality of the Islamic patterns on Ahlat tombstones (dates back to 12th-15th centuries) can be seen as relief. It can be said that the reason why these patterns can be seen as relief, is due to the material used. As can be imagined, the three-dimensionality can be given to the pattern by carving stones.

Three-dimensionality in Islamic patterns is in the form of overlapping. The overlaps are defined in a very regular and systematic way. To understand this order, we can consider one of the strips that form stars in Islamic patterns. If a strip passes under next strip, it will pass over another strip in the next encounter. This system repeats and a network-like tessellation is formed (Agirbas, 2020). An example of this type of pattern can be seen in Figure 2. Schneider (1980) has clearly described this situation in the drawings of Islamic geometric patterns in his book. In the drawings, although a pattern was created with a single line place to place, Schneider (1980) made the system in the pattern understandable by showing the lines passing under the other lines (strips) with a dashed line (Figure 3).

When we consider the patterns in three dimensions, we can easily say that the patterns consist of many pieces. The systematic formation of many pieces by overlapping is similar to the principle of forming reciprocal structure.

METHODOLOGY

In this study, six-pointed star polygon pattern tessellation in Schneider's (1980) book was used in lifting Islamic geometric patterns to the third dimension and transforming them into a reciprocal frame structure. The selected six-pointed star polygon pattern can be seen in the upper left part of Figure 3. In Schneider's (1980) drawings, it is determined how the third dimension of Islamic patterns is or could be (the stripes pass from the bottom or the top). This is important for the organization of the reciprocal frame structure.

The selected six-pointed star polygon pattern can be created with using reference circle units. The units can be multiplied and patterns with multiple units can be formed. In this study a pattern with 7 units, a pattern with 19 units and a pattern with 37 units have been created (Table 1, Figure 4).

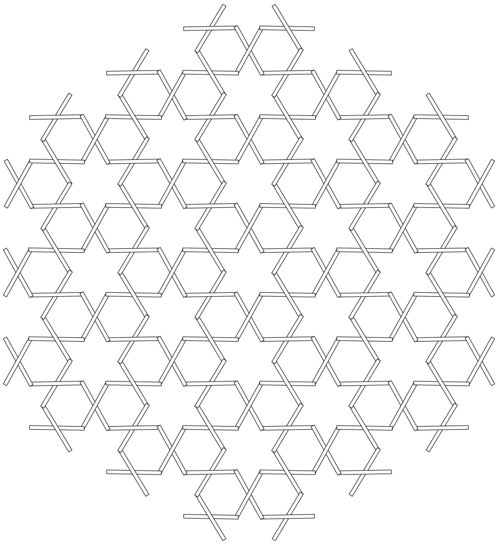


Figure 4
Pattern with 37 units

After creating three-dimensional models in Grasshopper [1], structural performance analyzes of the models were made using the Karamba [2] program. Karamba is a structural analysis program that works as an add-on to Grasshopper. Comparisons of the modelled structures were made by using different materials (timber, steel, concrete) and different number of supports were used in order to see structural performance difference.

Set-up

The patterns modelled in Grasshopper were given as input to the structural analysis script. For this, the "LineToBeam" component was used and it was connected to the "Assemble" component. "LineToBeam" component can transform lines defined in Grasshopper into beams. With this component, the star arms in the Islamic geometric pattern were defined as beams.

In addition, extra intermediary columns have been defined in order to ensure load transfer at the joints of the star arms. The heights of the intermediary columns were defined as 2 cm. These were also defined as beams to the script. This method is the method used by Gustafsson (2016) in his reciprocal structure study using Karamba.

In this study, the load transfers at the points, where the beams overlap at their center points, were excluded. No intermediary columns were added to these points.

With the "support" component, how the structure will be supported was defined. Different numbers of supports were added to the modelled pattern and various tests were carried out. The translational degrees of freedom (Tx, Ty and Tz) and the rotational degrees of freedom (Rx, Ry and Rz) of the supports were locked. Various constants were determined for the tests. These are the force of gravity and beam section type (rectangular, 3x 2 cm).

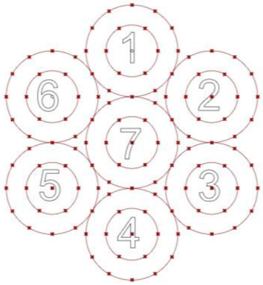
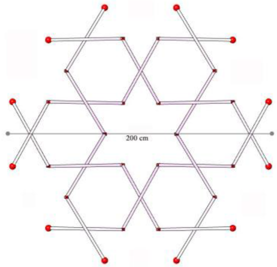
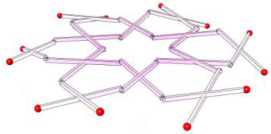
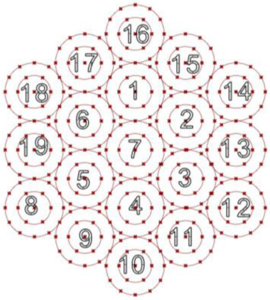
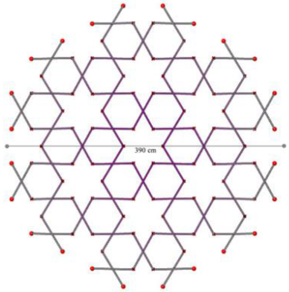
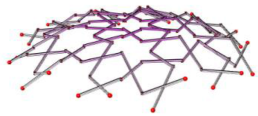
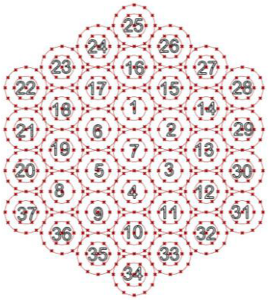
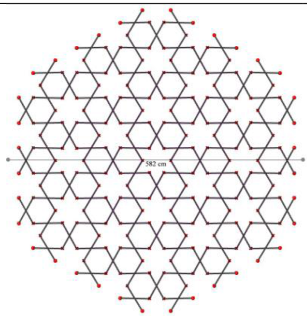
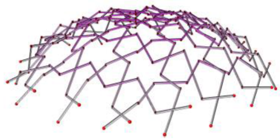
RESULTS

Results of the pattern with 7 units

The pattern with 7 units is 200 cm in diameter. 4 tests were conducted for structural performance analysis. The constants in these tests are given in Table 2. The variables in these tests are the number and location of the supports. In addition, structural performance simulations were performed with 3 types of materials, namely timber, steel and concrete, and the results were compared (Table 2, Table 3).

- In the tests for the pattern with 7 units, it can be seen that there is a little displacement value in all results. In all tests, it was seen that steel showed less displacement value than the other materials. However, it should be noted that this system is a very small system of 2 meters in diameter.
- In the analyses where the material is kept constant in the tests and the number of supports (24, 12 and 6 pieces) is used as a variable, we naturally see that the maximum displacement result increases as the number of supports decreases.

Table 1
The reference circles and the patterns

	Reference circles	Pattern	Pattern perspective
A pattern with 7 units	 Reference circles	 Plan view	 Perspective
A pattern with 19 units	 Reference circles	 Plan view	 Perspective
A pattern with 37 units	 Reference circles	 Plan view	 Perspective

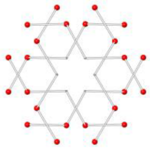
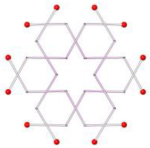
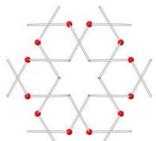
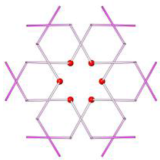
Test no	Support information	Constants	Results	Timber	Steel	Concrete
1	24 pieces  (Timber preview)	The force of gravity Rectangular cross section Profile dimensions (3x2 cm) Model diameter (200 cm)	Max displacement (cm)	0.007401	0.004708	0.009959
			Max compression (kN)	0.002076	0.027172	0.008656
			Max tension (kN)	0.000164	0.002153	0.000688
			Max moment (kNm)	0.000526	0.006795	0.002146
			Max shear (kN)	0.002127	0.027819	0.008857
2	12 pieces  (Timber preview)	The force of gravity Rectangular cross section Profile dimensions (3x2 cm) Model diameter (200 cm)	Max displacement (cm)	0.06949	0.042582	0.087929
			Max compression (kN)	0.002131	0.027893	0.008886
			Max tension (kN)	0.002118	0.027703	0.008821
			Max moment (kNm)	0.001525	0.019874	0.006313
			Max shear (kN)	0.004189	0.054786	0.017444
3	12 pieces  (Timber preview)	The force of gravity Rectangular cross section Profile dimensions (3x2 cm) Model diameter (200 cm)	Max displacement (cm)	0.013037	0.008526	0.018392
			Max compression (kN)	0.002152	0.028149	0.008965
			Max tension (kN)	0.00054	0.007065	0.00225
			Max moment (kNm)	0.00058	0.007589	0.002417
			Max shear (kN)	0.002127	0.027819	0.008857
4	6 pieces  (Timber preview)	The force of gravity Rectangular cross section Profile dimensions (3x2 cm) Model diameter (200 cm)	Max displacement (cm)	0.186463	0.116226	0.24289
			Max compression (kN)	0.004175	0.054618	0.017394
			Max tension (kN)	0.002092	0.027368	0.008716
			Max moment (kNm)	0.002052	0.026853	0.008552
			Max shear (kN)	0.004154	0.054348	0.017308

Table 2
The results of the tests made with the pattern with 7 units

- In each test, the results of maximum compression, maximum tension, maximum moment and maximum shear were the lowest in the timber systems.

Results of the pattern with 19 units

The pattern with 19 units is 390 cm meters in diameter. 2 sets of tests were conducted for the structural performance analysis. In the first set of tests, timber, steel and concrete were used as variables (Table 4). According to the results of these tests:

- The results of maximum displacement values were between 0.29 cm and 0.61 cm.
- It was seen that steel showed less displacement value than the other materials.
- The results of maximum compression, maximum tension, maximum moment and maximum shear were the lowest in the timber sys-

Table 3
Graphic representation of the tests (timber)

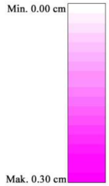
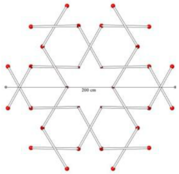
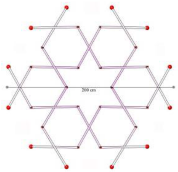
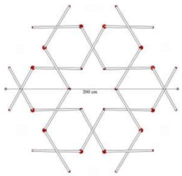
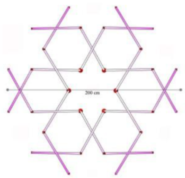
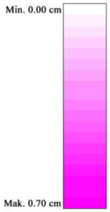
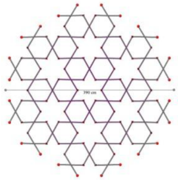
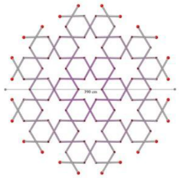
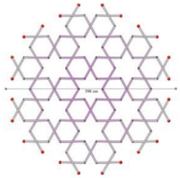
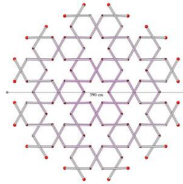
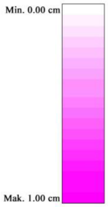
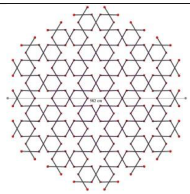
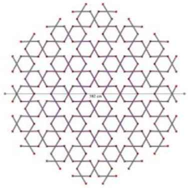
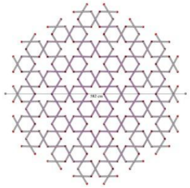
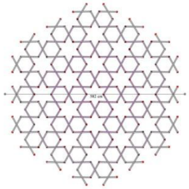
Units	Max Disp. scala	Graphic representation of test reults			
7 units		 3 x 2 cm cross-section 24 supports	 3 x 2 cm cross-section 12 supports	 3 x 2 cm cross-section 12 supports	 3 x 2 cm cross-section 6 supports
19 units		 3 x 2 cm cross-section 24 supports	 3 x 3 cm cross-section 24 supports	 3 x 4 cm cross-section 24 supports	 4 x 4 cm cross-section 24 supports
37 units		 3 x 2 cm cross-section 36 supports	 3 x 3 cm cross-section 36 supports	 3 x 4 cm cross-section 36 supports	 4 x 4 cm cross-section 36 supports

Table 4
The results of the material-based tests made with the pattern with 19 units

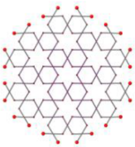
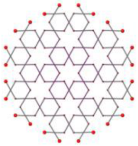
Support Information	Constants	Results			
			Timber	Steel	Concrete
24 pieces  (Timber preview)	The force of gravity Rectangular cross section Profile dimensions: 3x2 cm Model diameter (390 cm)	Max displacement (cm)	0.46251	0.29272	0.617727
		Max compression (kN)	0.012285	0.159323	0.050443
		Max tension (kN)	0.006408	0.083773	0.026664
		Max moment (kNm)	0.003636	0.047723	0.015233
		Max shear (kN)	0.0101	0.130222	0.041061

Table 5
The results of the profile cross-section based tests made with the pattern with 19 units (timber material)

Support Information	Constants	Results					
			3x2 cm	2x4 cm	3x3 cm	3x4 cm	4x4cm
24 pieces  (3x2 cm cross-section profile preview)	The force of gravity Material: Timber Model diameter (390 cm)	Max displacement (cm)	0.46251	0.311492	0.273919	0.199791	0.155149
		Max compression (kN)	0.012285	0.020553	0.020502	0.029042	0.036381
		Max tension (kN)	0.006408	0.00864	0.009659	0.012921	0.017171
		Max moment (kNm)	0.003636	0.004807	0.00527	0.007101	0.009364
		Max shear (kN)	0.0101	0.019468	0.018121	0.026615	0.032118

tems.

- In the comparison of the 7-unit (200 cm) and 19-unit (390 cm) models, the larger the diameter of the model gives the larger the maximum displacement value.

In the second set of tests, profile cross-section was used as a variable. Various dimensions (3x2 cm, 2x4 cm, 3x3 cm, 3x4 cm and 4x4 cm) for the cross-sections were used (Table 5). The material was fixed as timber. According to the results of these tests:

- As the dimension of cross-section increases, the maximum displacement value decreases. While the highest result was in the test with 3x2 profile cross-section, the lowest result was in the test with 4x4cm profile cross-section.

Results of the pattern with 37 units

The pattern with 37 units was 582 cm meters in diameter. 2 sets of tests were conducted for structural performance analysis. In the first set of tests, timber, steel and concrete were used as variables (Table 6). According to the results of these tests:

- The results of maximum displacement values were between 0.48 cm and 1.02 cm.
- It was seen that steel showed less displacement value than the other materials.
- The results of maximum compression, maximum tension, maximum moment and maximum shear were the lowest in the timber systems.
- In the comparison of the 19-unit (390 cm) and 37-unit (582 cm) models, the larger the diame-

Table 6
The results of the material-based tests made with the pattern with 37 units

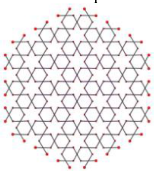
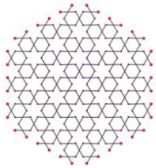
Support Information	Constants	Results			
			Timber	Steel	Concrete
36 pieces  (Timber preview)	The force of gravity Rectangular cross section Profile dimensions: 3x2 cm Model diameter (582 cm)	Max displacement (cm)	0.753151	0.482853	1.02794
		Max compression (kN)	0.018483	0.241312	0.07674
		Max tension (kN)	0.010988	0.143725	0.021617
		Max moment (kNm)	0.00516	0.067727	0.021617
		Max shear (kN)	0.015422	0.2016	0.064168

Table 7
The results of the profile cross-section based tests made with the pattern with 37 units (timber material)

Support Information	Constants	Results					
			3x2 cm	2x4 cm	3x3 cm	3x4 cm	4x4cm
36 pieces  (3x2 cm cross-section profile preview)	The force of gravity Material: Timber Model diameter (582 cm)	Max displacement (cm)	0.753151	0.518884	0.413456	0.298406	0.234384
		Max compression (kN)	0.018483	0.025739	0.028342	0.038217	0.050369
		Max tension (kN)	0.010988	0.014672	0.016495	0.022002	0.029324
		Max moment (kNm)	0.00516	0.006007	0.007259	0.009336	0.012916
		Max shear (kN)	0.015422	0.020977	0.023368	0.031321	0.041538

ter of the model gives the larger the maximum displacement value.

In the second set of tests, profile cross-section was used as a variable. Various dimensions (3x2 cm, 2x4 cm, 3x3 cm, 3x4 cm and 4x4 cm) for the cross-sections were used (Table 7).The material was fixed as timber. According to the results of these tests:

- As the dimension of cross-section increases, the maximum displacement value decreases. While the highest result was in the test with 3x2 profile cross-section, the lowest result was in the test with 4x4cm profile cross-section.

CONCLUSION

In this study, based on Islamic geometric pattern, reciprocal frame structure was created. This study was carried out with the selected Islamic geometric pattern tessellation. The pattern created with 7 units has been lifted from the 2nd dimension to the 3rd dimension. Afterwards, reference circles were systematically added around this pattern, and patterns with 19 and 37 units were created. Throughout this replication process, the principle of reciprocal frame structures has been adhered to. It has been observed that the system has a dome-like shape.

When the system was enlarged with increasing the units, it was observed that the maximum displacement values in the system normally increased. It was also seen that steel showed less displacement value than the other used materials in every tests.

In each test, the results of maximum compression, maximum tension, maximum moment and maximum shear were found to be the lowest in the timber systems.

There are many similar types, more complex or simpler Islamic geometric patterns in the literature. Similar studies can be performed using various Islamic geometric patterns. Thus, new ones can be added to the reciprocal frame structure morphologies.

According to the results of the structural performance analyses, the created reciprocal structure works effectively. As a limitation of this study, it should be noted that, the load transfers at the points, where the beams overlap at their center points, were not included in the structural performance analysis. In the case of inclusion of the load transfer at those points, the system probably will be more rigid. Also, various dimensions of the cross-sections should be tested.

It should be note that this is a simulation-based case study and may have intrinsic limitations and also may have biases based on the tool and method.

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