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CONTENTS

Monday, 11 June 2018

Perspective, Space, Dimension, Geometry

- 13 CORNELIE LEOPOLD. The Development of the Geometric Concept of Relief Perspective
- 19 AGOSTINO DE ROSA, ALESSIO BORTOT. Hunched Curves in the Vatican: The Vestibule Arch of the Pio Clementino Museum, Between Stereotomy and Anamorphosis
- 25 JOÃO PAULO CABELEIRA. Deconstructing the Imaginary Space of a Quadratura
- 31 GIUSEPPE D'ACUNTO. Notes on Oblique Space
- 37 SNEZANA LAWRENCE. Lost and Found: Some Mathematical Messages from Renaissance Tuscany to a 21st-Century Teacher
- 43 ALESSANDRA CAPANNA. The Four-Dimensional House Theorem
- 49 COSIMO MONTELEONE. The Mathematical Space of Daniele Barbaro
- 55 LAURA CARLEVARIS. N-Dimensional Perspective: The Mathematics behind the Interpretation of Ancient Perspective
- 61 STEFANO CHIARENZA. Peter Nicholson and the First Interpretation of Greek Architectural Mouldings as Sections of a Cone
- 67 RIZAL MUSLIMIN. A Grammatical Investigation of Utzon's Spherical Schema Evolution
- 73 ANTONIA REDONDO BUITRAGO. On Polygons, Set Squares and Mudéjar Carpentry
- 79 RADOSLAV ZUK. The Visible and Invisible Geometries of Venice

Tuesday, 12 June 2018

Historical Analysis

- 87 FRANCESCA FATTA, DOMENICO MEDIATI. The Design of Roman Mosaics in North Africa and their Geometric References
- 93 ASLI AGIRBAS. Algorithmic Decomposition of Geometric Islamic Patterns: A Case Study with Star Polygon Design in the Tombstones of Ahlat
- 99 BERNARD PARZYSZ. What We Can Learn from Roman Geometric Mosaics about the People Who Made Them
- 105 JOHN KENDALL HOPKINS. A Unified Schema of the Façade of San Miniato Al Monte: The Simultaneity of Interlocking Symbolic Harmonic, Irrational and Perfect Numbers
- 111 SANAZ AHMADZADEH SIYAHROOD, ARGHAVAN EBRAHIMI, MOHAMMADJAVAD MAHDAVINEJAD. Application Of Cubit-Gaz and Shape Grammar in Architecture Plan Design
- 117 ATHANASSIOS ECONOMOU. The Six Vitruvian Principles of Architectural Design Reframed within Contemporary Computational Design Discourse

- 123 ROBERTA SPALLONE, MARCO VITALI. *Regola* and *Licentia* in the *Extraordinario Libro* by Sebastiano Serlio
- 129 ORNELLA ZERLENGA, VINCENZO CIRILLO. *Della pianta delle Scale* of Guarino Guarini
- 135 ANNA MAROTTA, URSULA ZICH, MARTINO PAVIGNANO. Theories and Approaches in Fortifications Design: The Figure of Gaspare Beretta
- 141 STEFANO BRUSAPORCI, PAMELA MAIEZZA, GIANFRANCO RUGGIERI. A Reflection on the Cistercian *Bernhardinischer Grundtypus*
- 147 DANIELE CALISI, MATTEO MOLINARI. Giuseppe Valadier's Urban Layout for Piazza del Popolo in Rome
- 153 ISABELLA FRISO, ANDREA GIORDANO. The Design Process in the Salk Institute by Louis I. Kahn
- 159 VINCENZO BAGNOLO, ANDREA PIRINU, MARCELLO SCHIRRU. Geometrical Design Algorithms in Nineteenth-Century Prisons: The case of the *Rotunda* in Tempio Pausania
- 165 JOÃO PEDRO MARQUE SÊCO DIAS CARMONA. Urban Morphology of Geometric Pattern in the Villa Imperial de Petropolis
- 171 ARTURO GALLOZZI, MICHELA CIGOLA. Considerations on the Representations of the Analemma in Renaissance Editions of Vitruvius's *De Architectura*

Wednesday, 13 June 2018

Contemporary Analysis, Structures, Techniques of Design, Algorithms, Rule-Based Design

- 179 DENISE ULIVIERI, LUCIA GIORGETTI, BENEDETTA TOGNETTI. Vittorio Giorgini Spatiology-Morphology Architect: From 'Curved Systems' to 'Conventional Systems'
- 185 MICHAEL J. OSTWALD, MICHAEL J. DAWES. An Isovist Analysis of Frank Lloyd Wright's Hollyhock House
- 191 MICHAEL C. DUDDY. Logical Accidents: The Problem of the Inside Corner
- 197 ORIEL E.C. PRIZEMAN, CAMILLA PEZZICA, MARIANGELA PARISI, CLARALARISSA LORENZ. Function Should Follow Form: Futures for the Radiant Logic of Carnegie Public Libraries
- 203 SHEN GUAN SHIH, YI FENG CHANG. Composite Interlocking Structures of *SL* Strands
- 209 ASSUNTA PELLICCIO, MARCO SACCUCCI, ERNESTO GRANDE. A Key Nexus for Vault Systems from Lecce: Stereometric Correlation Between Shape and Structure
- 215 VALENTINA BEATINI. Morphology of Kinetic Structures by Means of Bar and Plate 4R-Linkages
- 221 MARCO HEMMERLING, CARLO DE FALCO. ArchiFold: An Educational Approach for the Integration of Mathematics in the Architecture Curriculum

- 227 MASSIMILIANO LO TURCO, URSULA ZICH, MARCO TRISCIUOGLIO, MICHELA BAROSIO, MARIA LUISA SPREAFICO, YOSEPH BAUSOLA PAGLIERO. Algorithmic Modeling and Design of the Architectural Shape: A Didactic Experience
- 233 FABIO BIANCONI, MARCO FILIPPUCCI, LORENZO CICULI. The Form of Music: Experiments between Cymatics and Engineering
- 239 MARCO CARPICECI, FABIO COLONNESE, Toward an Algorithm of Visual Design: The Mathematical Approach of Hermann Maertens' *Optische-Massstab*
- 245 CETTINA SANTAGATI, FEDERICO MARIO LA RUSSA, MARIATERESA GALIZIA, EUGENIO MAGNANO DI SAN LIO. Towards a Generic Parametric Algorithm for the Geometric Investigation of Baroque Oval Plans: An Application on Sicilian Cases
- 251 MANUEL ALEJANDRO RÓDENAS-LÓPEZ, PEDRO GARCÍA MARTÍNEZ, PEDRO MIGUEL JIMÉNEZ-VICARIO, ADOLFO PÉREZ EGEA, MARTINO PEÑA FERNÁNDEZ-SERRANO. Parametric Design Applied to Analysis and Optimization of Spatial Deployable Structures
- 257 MARIE-PASCALE CORCUFF. Jules Bourgoïn (1838-1908): A Forerunner of Generative Shape Grammars
- 263 BENAY GÜRSOY, MINE ÖZKAR. *Material Shapes* and How to Compute with Them
- 269 MANUELA BASSETTA. Form–Formal making of Traditional Chinese Architecture

Thursday, 14 June 2018

Ph.D. Day

- 277 PAOLO BORIN. Geometry, Science and Meaning in the Work of Guarino Guarini
- 281 CRISTIAN BOSCARO. *La Manière Universelle* of Girard Desargues for the Understanding of Stereotomic Structures
- 285 ALEXANDRA CASTRO. The Curve in the Architecture of Herzog & de Meuron
- 289 RAFFAELLA DE MARCO. Shapes and Models: The Survey for the Study of Structures in Historic Buildings
- 293 PAOLO DI PIETRO MARTINELLI. The Control of Illusory Space: The Contribution of Jacopo Barozzi da Vignola and the Anteroom of the Council in Palazzo Farnese at Caprarola
- 297 MARYAM DOROUDIAN, MOHAMAD REZA BEMANIAN, MOHAMAD JAVAD MAHDAVINEJAD. Exploring of Topological Architecture: a Review of Topology Influence on Architecture
- 301 WILLEM GYTHIEL, MATTIAS SCHEVENELS, DIRK HUYLEBROUCK. Generating Geodesic Grid Structures by Equally Subdividing Spherical Arc Segments
- 305 DIOGO PEREIRA HENRIQUES. Envisioning Future Public Spaces: Experiments in Co-Creation and Evaluation of Urban Visions

- 309 STEFANIA LANDI, ORIOL DOMÍNGUEZ MARTÍNEZ. Modularity in Ancient Grain Storage Systems: Historical Overview and In-Depth Analysis of Moroccan Fortified Granaries
- 313 ELISABETTA POZZOBON. Religious Architectural Heritage Losing Its Functions: Strategies to Mitigate the Problem and Provide New Value through Geographical Context Analysis
- 317 MAYCON SEDREZ. Complex or Complications? Fractal Geometry in Architecture and Urban Design
- 321 JAKUB ŚWIERZAWSKI. Curvilinearity in Architecture: Historical and Contemporary Ideas and Examples
- 325 VERA VIANA. Architectonic Tessellations as Constructive Modules

ALGORITHMIC DECOMPOSITION OF GEOMETRIC ISLAMIC PATTERNS: A CASE STUDY WITH STAR POLYGON DESIGN IN THE TOMBSTONES OF AHLAT

*Asli Agirbas*¹

Introduction

In this study, mathematics was used to analyze an existing monument. The mathematical design rules of the star polygon shape in the selected monument were determined based on the shape grammar theory. According to the shape grammar theory (Stiny and Gips, 1972; Duarte, 2005; Stiny, 2006; Ozkar, 2011; Knight and Stiny, 2015) (in other words rule-based design), forms are created using certain rules. The probable rules for the creation of the star polygon shape in this study were produced simultaneously in the computer environment using a visual programming language (in the Grasshopper program that works as a plug-in to the Rhino program) and a 3D algorithmic model of the shape was created.

The Research

The monument, a part of the pattern of which was analyzed, is a tombstone in the cemetery of Ahlat (Fig. 1). The cemetery of Ahlat is located in Ahlat, a district of Bitlis Province in Turkey's Eastern Anatolia Region. Ahlat hosted many different civilizations from the past to the present. The cemetery, which takes place in the tentative list of the World Heritage Convention of the United Nations Educational, Scientific and Cultural Organization (UNESCO), dates back to the 12th and 15th centuries. The historic area, now used as an open-air museum, is 200.000 square meters and contains more than 6000 tombstones. These tombstones, which have high-quality artistic values, contain carved designs that require great mastery. Although the tombstones are generally similar to each other in material, form, decoration and writing character, they are different from each other, especially ornamentation in detail. The tombstones, which are generally in the shape of rectangular, are 40-70 cm wide and 1-5 m high. The material used in these tombstones is ignimbrite, Ahlat's local construction material. It is locally called 'Ahlat Stone' (Isik et.al. 2015). Ignimbrite, a volcanic stone, is formed in the end of volcanic eruptions. It is a light and easily shaped stone. Thus, the masters could easily shape this stone (Baykara and Isik, 2016).

The greatest characteristic of the analyzed pattern in this study is that it is in the form of carving and is in a knotted structure. Therefore, when the rules of analysis for this pattern were done, the pattern was considered 3-dimensionally. In the pattern examined in this study, it is obvious that the star-shaped polygon takes place in a hexagon. However, unlike many other star-shaped Islamic patterns (Bodner, 2012; Sarhangi, 2012), the end points of this star do not intersect with the hexagon.

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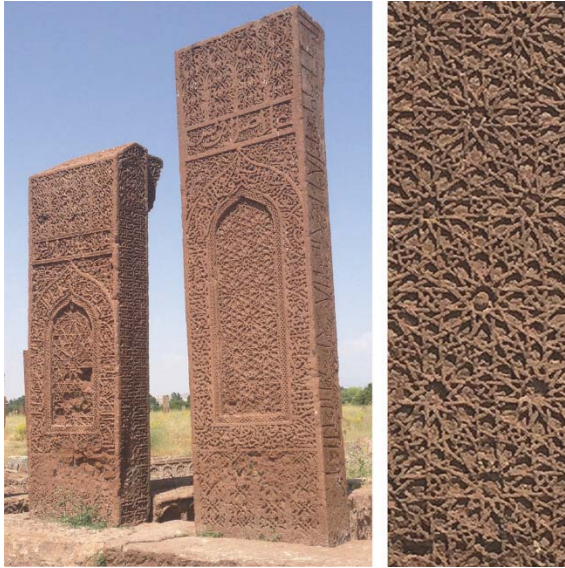


Fig. 1. Tombstones from Ahlat Cemetery and selected star-shaped pattern for the analysis

The parts, which intersect with hexagon, are the knots at the end points of the star. Therefore, while this star-shaped pattern was being analyzed in the hexagon, the star was handled with its knots in the end points. According to this, when the knotted star shape in a hexagon was analyzed as three-dimensionally, 3 L-shaped pieces knotted into each other were defined. And in the hexagon, 4 pieces of each L-shaped pieces were determined.

The possible series of rules for creating this pattern was determined (Fig. 2) and transferred to the algorithms in the same rules of framework (Fig. 3). Accordingly, a circle was first created for the creation of this pattern. The ‘circle’ component was used to create a circle in Grasshopper. The size of the circle can be parametrized by the ‘number slider’ parameter connected to the radius value of the circle component. Afterwards, this circle was divided into 6 equal pieces.

Six point outputs were obtained as a result of this process using ‘divide curve’ component. This value can easily be parametrized by the number slider placed in the ‘number of segments’ input of the divide curve component. A polygon (hexagon) was created with combining these points using ‘list item’ and ‘line’ components (Rule Series 1). Then, a circle was created in the centroid of the hexagon with the help of ‘area’ and ‘circle’ components. With the aid of a divide curve component, this inner circle was divided into 24 equal pieces. With the aid of the list item component, 3 points with equal distance were selected from the parts in $\frac{1}{4}$ of this inner circle (Rule Series 2).

Then, with the help of ‘evaluate curve’ component, 20%, 70% and 80% parts of one edge of the hexagon (that was created in Rule Serie 1), and 30% part of the other edge of the hexagon and 30% and 80% parts of another edge of the hexagon were determined.

Rule Series	Process				Overall view
RS 1					
RS 2					
RS 3					
RS 4					
RS 5					
RS 6 (3D)					
RS 7 (3D)					

Fig. 2. Set of rules for pattern analysis

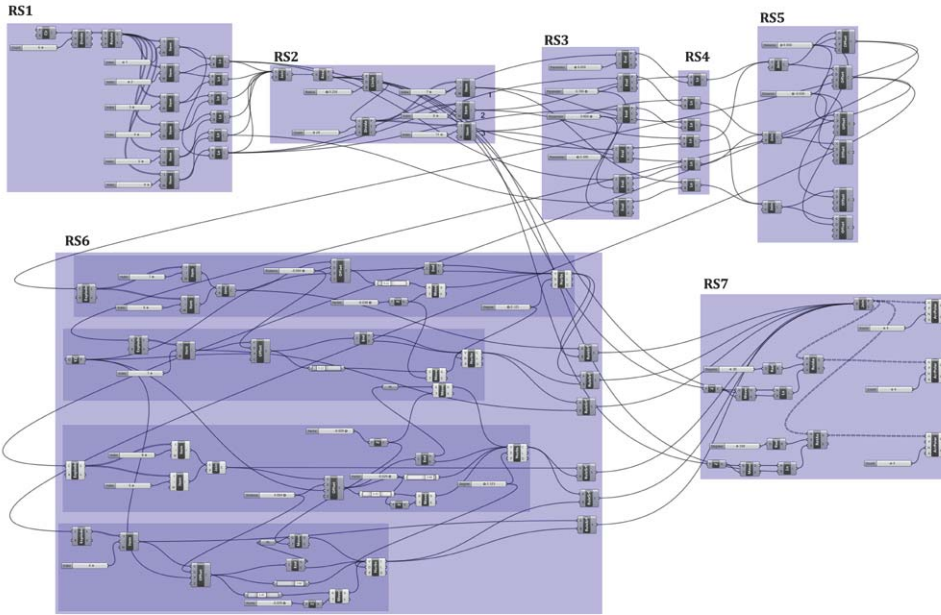


Fig. 3. Transfer of the rules to algorithm

In addition, these ratios can easily be parametrized by number sliders connected to the evaluate curve component (Rule Series 3). With the aid of the ‘line’ component, three types of L-shaped curve were created in 2 dimensions, combining the 3 points in the inner circle and the corresponding points on the edges of the hexagon (Rule Series 4). With the aid of the ‘offset curve’ component, the first L-shaped curve, the second L-shaped curve and the third L-shaped curve, which were created in 2D, were taken their offset to double side to create a base area for 3 dimensions. Again, with the number slider connected to the offset component, the offset value can easily be parametrized (Rule Series 5). At the next stage, the form was brought to 3D. The offsets of L-shaped curve in 2D were offset along Z axis to bring to the third dimension. Three points were determined above the last offset curve (along Z axis). From tree points, one point is the starting point of the curve, one is the ending point, and one is determined in between the points (somewhere between the start and end points). This midpoint is determined with the help of the ‘point on curve’ component and can be easily parameterized horizontally. Then, this point in the middle was moved up or down in the Z axis (vertically) according to the knot structure. Again, this ratio can easily be parametrized. Afterwards, a curve from these 3 points was obtained with the aid of the ‘nurbs curve’ component. The first created 2D curve and the curve created later in the third dimension were converted to the surface with the aid of the ‘ruled surface’ component. The other 2 L-shaped form was actually understood to be the rotated version of the first L-shaped form created. The interesting point here is that the mirror image look in 2D disappeared when the shape was analyzed in 3 dimensions. Generally, these types of Islamic patterns are shaped by taking the mirror images of the forms. However, in the analyzed shape of this study, it can be seen that the pattern was created with the circular repeat of L-shaped

form. Therefore, with the aid of the 'rotate axis' component, this 3-dimensional L-shaped form was rotated in such a way that it overlies the other 2 L-shaped curves seen in two dimensions (Rule Series 6). With the aid of the 'array polar' component, each 3-dimensional L-shaped form, individually, each to be 4 in total, was copied circularly (Rule Serie 7) (Fig. 4).

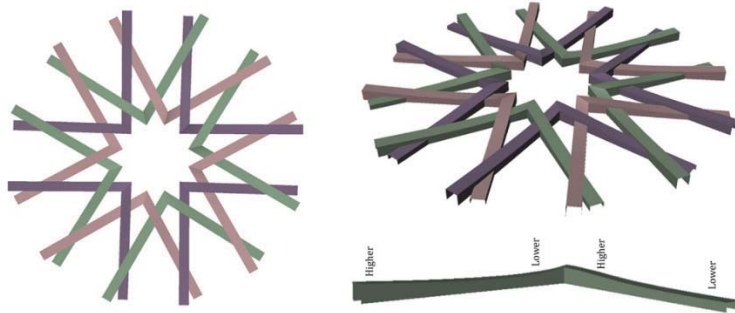


Fig. 4. A 3-dimensional shape of the pattern produced by algorithms

There is no single mathematical set of rules to create this type of forms. The same type of forms can be created with different rules. In addition, while creating the form with algorithms, the creation of rules in detail with well-defined steps provides great convenience for both the design analyzer and the designer. Because, in parametric definition, desired changes can easily be made by returning to a specific part. Along with this, the important benefit of parametric design tools is that the numerical values specified in the parametric definition can be changed very easily. Thus, many different variations, which can be done with the same rules series, can be created easily.

The 3-D form produced in this study can be transformed into a complete mesh that passes each other by the same principle. In addition, as the next step in this study, the knot details of the interlocking star shapes can also be analyzed mathematically and transferred into algorithms

Conclusion

Generally, geometric patterns are analyzed by specifying shape grammar rules. The next step of this, again under shape grammar theory, with determining the rules of the form, it may be analyzed based on algorithms. The use of constructivist methodology with well-defined steps, could be one of the answers to an up-to-date problem of designers' reasoning on dataflow programming. Because, the rules become break points in the definitions, and these break points form the backbone of the designs. As in this study, the patterns, which were transferred into algorithms by using parametric design tools, can be easily modified by changing the parameters in the algorithm and this can create an infrastructure for the future designs. With this method, for the artifacts containing various patterns that cannot be read, various variations can be produced in a short time and many alternatives can be obtained.

Various complex forms containing knots can be analyzed by algorithm-based software more easily, especially in 3D.

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